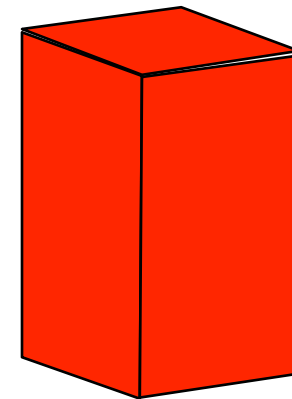
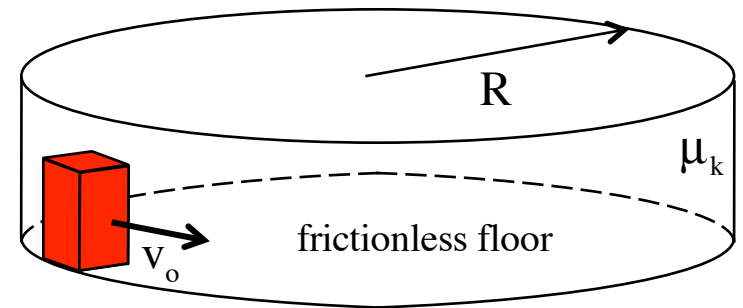


## Added Thrill Problem—The Ring

Securely fixed on a frictionless surface is a frictional, circular wall (coefficient of friction  $\mu_k$ ) of radius “R.” A mass “m” is given an initial speed of  $v_o$  moving around this wall. How long will it take for the mass to reach a speed of  $v_o/2$ ?

This is an old-school AP question, and it's nice because the centripetal force acting varies in magnitude as the body slows due to friction. So how do you deal with it? You have to be clever. See if you can do so before looking at the solution. (Hint: You might want to start with a f.b.d.)

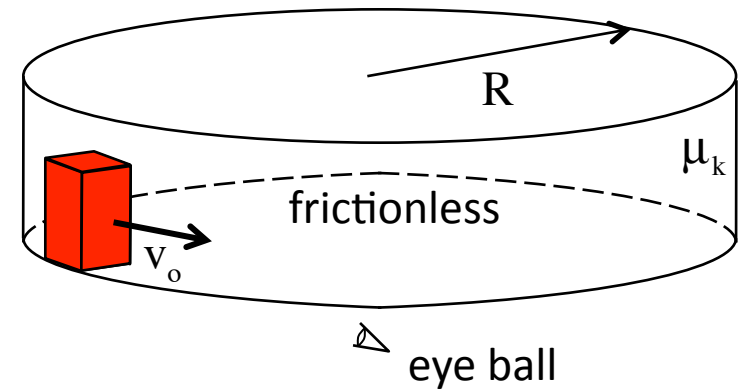
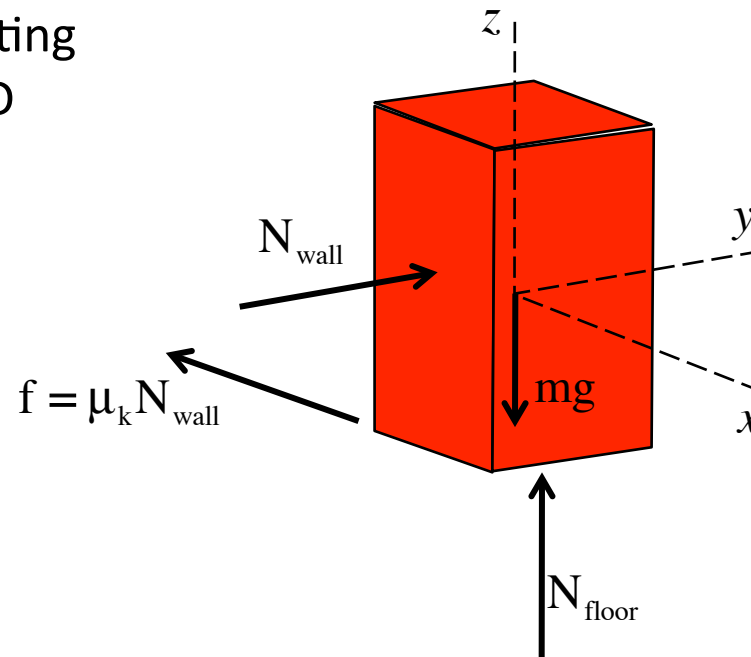




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In fact, this is a N.S.L. problem. Presenting a f.b.d. from a 3-D view, we get:

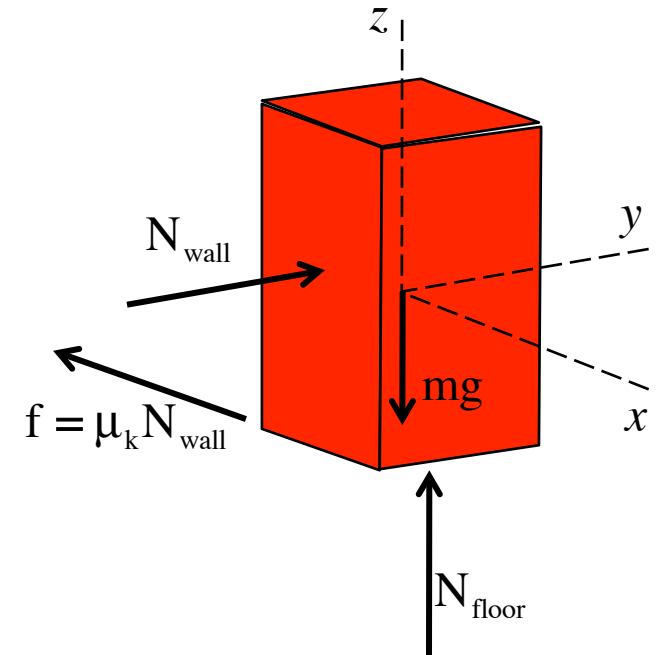


The floor's *normal force* does nothing as the floor is frictionless, but the wall's *normal force* is important because it not only provides the centripetal force needed to motivate the body along a curved path, it is also the quantity that determines the accelerating *frictional force* in the *negative x-direction*.

We need relationships, so using N.S.L. in various directions in no particular order:

$$\begin{aligned} \underline{\sum F_y} : \\ N_{\text{wall}} &= ma_c \\ \Rightarrow N_{\text{wall}} &= m \frac{(v)^2}{R} \end{aligned}$$

Minor note for the next part: Whenever you find yourself in a situation in which you want to manipulate Newton's Second Law to determine a time, you need to leave the "ma" part of N.S.L. in its most general form. That is, even if it is obvious that the acceleration is negative, you *don't unembed the negative sign* when you write out " $ma = m \left( \frac{dv}{dt} \right)$ ." With that in mind:



$$\underline{\sum F_x :}$$

$$-f_k = ma$$

$$\Rightarrow -\mu_k N_{\text{wall}} = m \frac{dv}{dt}$$

$$\Rightarrow -\mu_k \left( m \frac{v^2}{R} \right) = m \frac{dv}{dt}$$

$$\Rightarrow -\left( \frac{\mu_k}{R} \right) dt = \frac{dv}{v^2}$$

$$\Rightarrow -\left( \frac{\mu_k}{R} \right) \int_{t=0}^t dt = \int_{v_0}^{v_0/2} \left( \frac{1}{v^2} \right) dv$$

$$\Rightarrow -\left( \frac{\mu_k}{R} \right) t = \left( -\frac{1}{v} \right) \Big|_{v_0}^{v_0/2} = \left( -\frac{1}{v_0/2} \right) - \left( -\frac{1}{v_0} \right) = \left( -\frac{2}{v_0} \right) + \left( \frac{1}{v_0} \right) = \left( -\frac{1}{v_0} \right)$$

$$\Rightarrow -\left( \frac{\mu_k}{R} \right) t = -\frac{1}{v_0}$$

$$\Rightarrow t = \frac{\mu_k R}{v_0}$$

